Epistemic Utility Theory of Legal Belief

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Theories of Legal Proof

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Legal Belief based on Credence

(Q) How should we form legal beliefs based on various kinds of evidence?

- “Legal beliefs” should be more cautious than rational beliefs in everyday reasoning.
- Thus, legal beliefs should be stronger than, for example, acceptance, which might be not truth-conducive.
- Our best evidence might not guarantee access to the truth, although we endeavor to form maximally accurate beliefs.
- I will suggest a way of forming legal beliefs that are stronger than acceptance or rational beliefs in everyday contexts, but weaker than knowledge.
- In my view, Leitgeb’s stability theory of belief (Leitgeb, 2017) and Williamson’s concept of establishing (Williamson, 2020) also can be viewed as modeling intermediate attitudes between rational beliefs and knowledge.

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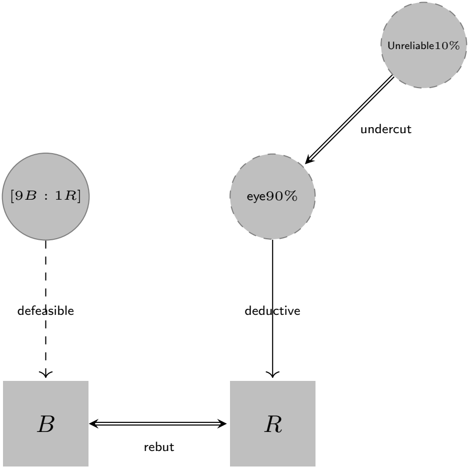
Motivation

(Q) What if some statistical evidence and an eyewitness testimony support conflicting conclusions?

– Using formal argumentation framework:



(A) The statistical evidence seems weaker than the eyewitness testimony.

Observations

There were 10 prisoners. The guard was murdered. G_i means "i is guilty".

(Case 1) (Statistical evidence) 9 prisoners collectively murdered the guard;

$\Pi = \{\neg G_1, \neg G_2, ..., \neg G_{10}\}$; $P(G_i) = 0.9$ for all i

(Case 2) (Eye-witness) One witnessed that i murdered the guard, and the reliability of the witness is 0.9; $\Pi = \{G_i, \neg G_i\}$; $P(G_i) = 0.9$

(1) (Source of belief) The source of belief in Case 1 is the fact that 9 prisoners collectively murdered the guard while the source of reliability in Case 2 is e.g. the sample data generated by i.i.d experiments.

(2) (Dependency) In Case 1 G_i and G_j are dependent events; $P(G_i \text{ and } G_j) = 1 - (P(\neg G_i) + P(\neg G_j)) = 0.8$. In contrast, if there is another witness for j in Case 2, G_i and G_j are independent; $P(G_i \text{ and } G_j) = P(G_i)P(G_j)) = 0.81$.

(3) (Belief update) Suppose that we get the information that the 10th prisoner is guilty. In Case 1, we should revise our credence as follows: $P(G_i) = \frac{8}{9}$ for $i \in \{1, ..., 9\}$. In Case 2, we are not required to revise our credence unless the prisoner in question is the 10th prisoner.

(4) (Resiliency) (3) shows that the evidence of Case 2 is more resilient than the one of Case 1.

Weight of Evidence

My claim: statistical evidence is weaker evidence than eye-witness in terms of weights of evidence.

cf. The balance of evidence: a matter of how decisively the data tells for or against the hypothesis (Joyce, 2005) This can be measured by means of confirmation measures.

- The weight of evidence: a matter of gross amount of data available (Joyce, 2005)
- The weight of evidence might be measured by means of imprecise probabilities, second-order probability or resiliency.

Comparison of the wight of evidence

	10000 i.i.d. random experiments	
$\succeq$	100 i.i.d. random experiments	eye-witness
$\succeq$	1-shot random experiments	100 lotteries
$\succeq$	1-shot quasi-random experiments	100 prisoners
$\succeq$	enumerative induction based on 1-shot quasi-random experiments	iphone theft
$\succeq$	uniform distribution based on total ignorance	

Desiderata of Legal Belief based on Credence

- Deductive Cogency: Deductive Closure and Consistency
  (Need to avoid the Lottery Paradox → Relax the Lockean Thesis → Holistic Approaches)
- Truth-Conduciveness: Maximizing Expected Accuracy
  (Are there more norms for the Epistemic Utility Function? To be discussed later.)
- Are there more desiderata for legal beliefs? To be discussed later.

Theories of Cogent Beliefs based on Credence in the Literature

Belief Binarization	Cogent Beliefs ought to
Leitgeb's Stability Theory (ST)	have stably high probability
Lin & Kelly's Tracking Theory	track Bayesian conditioning
Goodman & Sallow's Normality Theory	have the comparative normality
Wang and Kim's Distance Minimization (DM)	approximate probability
Wang and Kim's Epistemic Utility Theory (EUT)	be expected to be accurate

(ST) $BelX$ iff $Pr(X|Y) > r$ for all Y such that $Pr(Y) > 0$ and $\neg Bel\neg X$

(DM) $BelX$ iff B implies X where B minimizes $D(Pr, Uni(B))$

(This rule satisfies suspension principle: if the credence is a uniform distribution on B, then the strongest believed proposition should be B.)

(EUT) $BelX$ iff B implies X where B maximizes $EU_{Pr}[B]$

(If we use proper scoring rules as utility then this rule is a distance minimization rule.)

(Q) A theory of Legal cogent Beliefs based on Credence?

(A) Stable Expected Utility Theory

Stability under Possible Future Evidence

(Q) Apart from truth-conduciveness and deductive cogency, are there other epistemic norms for legal beliefs?

(A) My answer is that beliefs ought to be stable under possible future relevant evidence.

- To identify possible future evidence, we ought to be able to construct a suitable space of evidence and set the level of cautiousness (we don't need to be concerned about Cartesian scenarios in the court). And we should consider all possible evidence that qualifies as possible future relevant evidence.
- We want to check if our inquiry is sufficiently settled to form a belief. So we should analyze how robust our credence function is against possible future evidence, since our collected evidence can be encoded in a credence function.

Stable Expected Utility Theory (Stable EUT)

[Step I] Suitable Space of Hypotheses and Evidence

Construct suitable space of hypothesis and evidence. This space should not be not arbitrary determined, but reflect on what relevant future possible evidence is like.

[Step II] Forming Credences: Imprecise Probability

Collected evidence does not give us precise probability. Beliefs should be stable under all the probabilities conditional on future possible evidence in the space of [Step I], which is likely to be obtained.

[Step III] Credences and stable and cogent Beliefs: Epistemic Utility Theory

Cogent beliefs should be determined by Epistemic Utility Theory and should be stable under all credences of [Step II].

Stable EUT - [Step I] Suitable Space of Hypotheses and Evidence

Construct a suitable space $\mathcal{E}$ of hypotheses and evidence:

(1) All logically related hypothesis $\mathcal{H}$ should be included.

(2) All information given in the collected evidence $\mathcal{D}$ should be included.

(3) All and only relevant possible future evidence $\mathcal{P}$ should be identified.

Example 1: Prisoners Case

Let G_i mean "i is guilty".

(1) (Statistical evidence) There are 10 prisoners; 9 prisoners collectively murdered the guard. $\Pi = \{\neg G_1, \neg G_2, ..., \neg G_{10}\}$; $P(G_i) = 0.9 \; \forall i \in \{1, ..., 10\}$

$\neg G_1$	$\neg G_2$	$\neg G_3$	$\neg G_4$	$\neg G_5$	$\neg G_6$	$\neg G_7$	$\neg G_8$	$\neg G_9$	$\neg G_{10}$

(2) (Eye-witness) There is an eye-witnesses whose reliability is 0.9 who witnessed 1 murdered the guard. $\Pi = \{G_1, \neg G_1\}$; $P(G_1) = 0.9$

$\neg G_1$	G_1

Example 2: Blue Buses and Red Buses

Let B_i mean "Blue bus driver i hit the pedestrian" and R mean "The red bus hit the pedestrian".

(1) (Statistical evidence) There are 10 buses; 9 of 10 belong to the blue bus company; 1 of 10 to the red bus company. $\Pi = \{B_1, ..., B_9, R\}$; $P(B_i) = 0.1 \ \forall i \in \{1, ..., 9\}$, $P(R) = 0.1$

(2) (Eye-witness) There is an eye-witnesses whose reliability is 0.9 who witnessed the red bus hit the pedestrian. $\Pi = \{R, \neg R\}$; $P(R) = 0.9$

Example 3: Merging two cases

Let S mean "John stole the car" and M mean "John murdered the CEO", respectively. Suppose that two cases are causally independent and so probabilistically independent. $\Pi_s = \{S, \neg S\}, P(S) = 0.8; \Pi_m = \{M, \neg M\}, P(M) = 0.7$; $\Pi_s \times \Pi_m$ is not a suitable space, because there would be no possible piece of evidence for the events "$\pm S$ and $\pm M$".

Stable EUT - [Step II] Forming Credences: Imprecise Probability

1. Point Credence for Reference based on collected evidence
   Let $P := P_{\mathcal{D}}$ be a Point Credence for Reference.
   We can use different rationality norms like the principal principle, the narrowest reference class, or the principle of indifference, or deference to the expert's opinion, etc. Here, we will not fix the set of rationality norms, which will be another story.

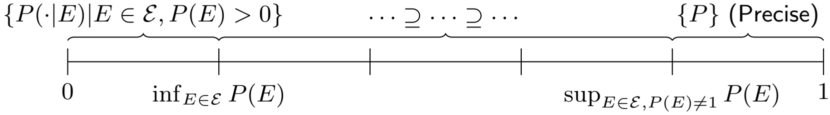
(Q) How can we represent the robustness against some possible future evidence?

2. Forming Imprecise Probability based on possible evidence

$$\mathbb{P} := \{P(\cdot|E) | E \in \mathcal{E}, P(E) > s\}$$

   How to select possible future evidence? Here, s is the selection criterion and represents degrees of stability. This value encodes how cautious or risk-averse we should be: The smaller s is, the more imprecise $\mathbb{P}$ is and the more cautious we are.

Degree of Stability



Observation

(1) If $s < s'$, then $\{P(\cdot|E)|E \in \mathcal{E}, P(E) > s\} \supseteq \{P(\cdot|E)|E \in \mathcal{E}, P(E) > s'\}$

(2) If $1 > s \geq \sup_{E\in\mathcal{E},P(E)\neq 1} P(E)$, $\mathbb{P}$ is $\{P\}$.

(3) If $s \leq \inf_{E\in\mathcal{E}} P(E)$, $\mathbb{P} = \{P(\cdot|E)|E \in \mathcal{E}, P(E) > 0\}$.

- Observation (1): The smaller the degree of stability, the more imprecise our credence becomes.
- Observation (2) indicates the degree of stability at which we can retain the point credence for reference.
- Observation (3) indicates the degree of stability at which our credence becomes maximally imprecise.

Example: Four Prisoners

$\neg G_1$	$\neg G_2$	$\neg G_3$	$\neg G_4$

- $W = \{\neg G_1, \neg G_2, \neg G_3, \neg G_4\}$
- $P(G_i) = 0.75 \; \forall i \in \{1, ..., 4\}$

$$\mathbb{P} = \{P(\cdot|E) | E \in \mathcal{E}, P(E) \geq 0.5\}$$
$$= \{P(\cdot|W), P(\cdot|G_1), P(\cdot|G_2), P(\cdot|G_3), P(\cdot|G_4),$$
$$P(\cdot|G_1 \cap G_2), P(\cdot|G_1 \cap G_3), P(\cdot|G_1 \cap G_4), P(\cdot|G_2 \cap G_3), P(\cdot|G_2 \cap G_4), P(\cdot|G_3 \cap G_4)$$

$$\begin{aligned}
P(G_1|W) &= 0.75 & P(G_1|G_1 \cap G_2) &= 1 \\
P(G_1|G_1) &= 1 & P(G_1|G_1 \cap G_3) &= 1 \\
P(G_1|G_2) &= 0.66 & P(G_1|G_1 \cap G_4) &= 1 \\
P(G_1|G_3) &= 0.66 & P(G_1|G_2 \cap G_3) &= 0.5 \\
P(G_1|G_4) &= 0.66 & P(G_1|G_2 \cap G_4) &= 0.5 \\
& & P(G_1|G_2 \cap G_4) &= 0.5
\end{aligned}$$

Example: Eyewitness

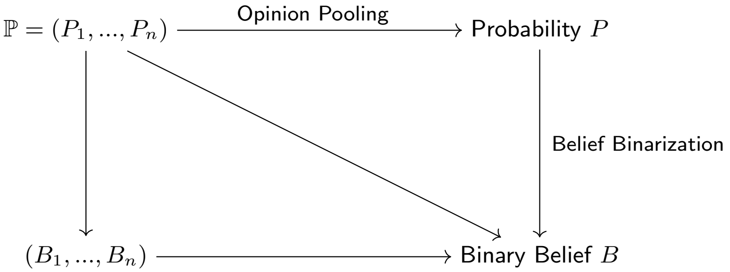
$\neg G_1$	G_1

- $W = \{\neg G_1, G_1\}$
- $P(G_1) = 0.75$

$$\mathbb{P} = \{P(\cdot|E)|E \in \mathcal{E}, P(E) \geq 0.5\} = \{P(\cdot|W), P(\cdot|G_1)\}$$

$$\begin{aligned} P(G_1|W) &= 0.75 \\ P(G_1|G_1) &= 1 \end{aligned}$$

Stable EUT - [Step III] Credences and Stable Cogent Beliefs : Epistemic Decision



Expected utility maximization given Imprecise Probability $\mathbb{P}$: $BelX$ iff $B \rightarrow X$ where

(1) Expected Utility Maximization w.r.t. pooled opinion $f(\mathbb{P})$:

B maximizes $EU_{f(\mathbb{P})}[B]$

(2) Maximize Minimum (worst case) Expected Utility:

B maximizes $min_{P \in \mathbb{P}} EU_P[B]$

(3) Expected Utility w.r.t. all probabilities in $\mathbb{P}$:

B maximizes $EU_P[B]$ for all $P \in \mathbb{P}$

Desiderata of Epistemic Utility (Accuracy) of binary belief

Example: $W := \{w_1, w_2, w_3\}$; assume that U is neutral (i.e., invariant under permutation between worlds) and $R_3 = 0$

B	$U_{w_1}(B)$	$U_{w_2}(B)$	$U_{w_3}(B)$	$EU_{Pr}[B]$	Conditions of B maximizing $EU_{Pr}[B]$
$\{w_1\}$	R_1	W_1	W_1	$R_1 p_1 + W_1(1-p_1)$	$p_1 \geq \frac{-W_1}{R_1 - W_1}$ and $p_1 \geq -\frac{R_2 - W_2}{R_1 - W_1} p_3 + \frac{R_2 - W_1}{R_1 - W_1}$
$\{w_1, w_2\}$	R_2	R_2	W_2	$R_2(1-p_3) + W_2 p_3$	$p_3 \leq \frac{R_2}{R_2 - W_2}$ and $p_1 \leq -\frac{R_2 - W_2}{R_1 - W_1} p_3 + \frac{R_2 - W_1}{R_1 - W_1}$
$\{w_1, w_2, w_3\}$	R_3	R_3	R_3	R_3	$p_1 \leq \frac{W_1}{R_1 - W_1}$ and $p_3 \geq \frac{R_2}{R_2 - W_2}$

- Principle of exclusion of prejudice: Neutrality
- Truth-directedness (TD): $R_i > W_j$ for all i, j
- Reward-informativeness (RI): $R_1 > R_2 > ... > R_{n-1} > R_n$
- The Suspension Principle: if the credence is a uniform distribution on B, then the strongest believed proposition should be B.

Thm1. Assume neutrality. The suspension principle implies (TD) and (RI).

Thm2. If we use proper scoring rules as utility then the EUM rule satisfies the suspension principle.

Example: Four Prisoners and Eyewitness

We want to select B that maximizes $EU_P[B]$ for all $P \in \mathbb{P}$.

(1) Consider $P(\cdot|G_3 \cap G_4) =: Q$. Let $w_1 = \neg G_1$ and $w_2 = \neg G_2$. $Q = (0.5, 0.5)$

$\neg G_1$	$\neg G_2$	$\neg G_3$	$\neg G_4$

Let U satisfy truth-directedness $(R_1 > R_2 = 0 > W_1)$ with $R_1 < |W_1|$.

$EU_Q(\{w_1\}) = 0.5R_1 + 0.5W_1 < 0$

$EU_Q(\{w_1, w_2\}) = 0$

$\{w_1, w_2\}$ maximizes $EU_Q[B]$.

Since $\{w_1, w_2\}$ does not imply G_1, G_1 is not believed.

(2) Remember $\mathbb{P} = \{P(\cdot|W)(= P), P(\cdot|G_1)(=: Q)\}$.

Let $w_1 = G_1$ and $w_2 = \neg G_1$. $P = (0.75, 0.25)$ and $Q = (1, 0)$

$\neg G_1$	G_1

Let U satisfy truth-directedness $(R_1 > R_2 = 0 > W_1)$ with $3R_1 > |W_1|$.

$EU_P(\{w_1\}) = 0.75R_1 + 0.25W_1 > 0$	$EU_Q(\{w_1\}) = 1$
$EU_P(\{w_1, w_2\}) = 0$	$EU_Q(\{w_1, w_2\}) = 0$

$\{w_1\}$ maximizes $EU_P[B]$ and $EU_Q[B]$ and so $B = \{w_1\}$.

Since B implies G_1, G_1 is believed.

Summary: Stable Expected Utility Theory

[Step I] Suitable space of hypotheses and evidence is given by context.

[Step II] Collected evidence, especially statistical evidence, does not give precise probability. Moreover, credences are formed by not only collected evidence but also possible future evidence determined by suitable space.

[Step III] Cogent beliefs are given by epistemic utility theory for belief binarization of imprecise probability.

Comparison with Other Approaches

- Leitgeb's Stability Theory of Beliefs demands stably high probability conditional on all not disbelieved propositions. This requires stability under epistemic possibility—possible according to the agent's beliefs—, which gives us a circular definition of beliefs. Agents in this theory can have only degrees of stability that are required to form cogent and Lockean beliefs.
- Williamson's Establishing uses the Lockean thesis, thus gives up deductive cogency.
- Stable EUT requires stability under relevant rebut possibility given by context. And this theory models any arbitrary degree of stability.

	Stability Theory of Belief	Establishing	Stable EUT
Deductive Cogency	O	X	O
Stability	O	O	O
Expected Accuracy Maximization	X	O	O

Further Research

- Measuring the weight of evidence is a tricky task. How do we compare the weight of evidence between:
  - One-shot quasi-random experiments with 100 individuals
  - Enumerative induction based on one-shot quasi-random experiments with 1000 individuals

  Devising a faithful quantitative measure of the weight of evidence will be interesting, and guide us toward a more refined way of forming credence, one that takes into account the varying weights of evidence.
- Additional desiderata for utility functions in legal reasoning can be considered. For instance, how do we encode the principle of presumption of innocence into a utility function?

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