

What's Wrong with Partial Punishment?

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Table 1: How to get to work?

	Not raining	Raining
Walking through the park	6	-2
Walking through the city	4	1
Taking the subway	2	2

If the probability of rain were, say, 60% then the expected utility of walking through the city would be 2.2, which exceeds that of walking through the park or taking the subway (1.2 and 2 respectively).

Table 2: To convict or acquit?

	Defendant is guilty	Defendant is innocent
Acquit	-5	0
Convict	5	-100

Given this assignment, the standard of proof should be around 0.91 – that is, convicting will maximise expected utility if the probability of guilt is greater than ≈ 0.91 and acquitting will maximise expected utility if the probability of guilt is lower than ≈ 0.91 . (And if the probability is exactly 0.91 – or more strictly exactly $0.9\overline{09}$ – then the expected utilities will be equal). More generally, if U_{ag} is the utility of acquitting the guilty, U_{ai} is the utility of acquitting the innocent, U_{cg} is the utility of convicting the guilty and U_{ci} is the utility of convicting the innocent, the guilt probability at which the expected utility of convicting overtakes that of acquitting is given by $1/(1 + ((U_{cg} - U_{ag})/(U_{ai} - U_{ci})))$.

Table 3: To blame or not to blame?

	Person is guilty	Person is innocent
Withhold blame	-2	0
Blame	2	-12

Given this assignment, blaming will maximise expected utility if the probability of guilt is greater than 0.75 and withholding blame will maximise expected utility if the probability of guilt is lower than 0.75. (And if the probability of guilt is exactly 0.75 then the expected utilities will be equal).

Table 4: To blame or not to blame? (partial blame included)

	Person is guilty	Person is innocent
Withhold blame	-2	0
Partially blame	$-2 < x < 2$	$-12 < y < 0$
Fully blame	2	-12

If, say, $x = 1$ and $y = -6$ and the probability of guilt is between ≈ 0.67 and ≈ 0.86 then the expected utility of partial blame him would be higher than no blame and full blame.

Table 5: To blame or not to blame? (multidimensional utilities)

	Person is guilty	Person is innocent
Withhold blame	$\langle 0, -1 \rangle$	$\langle 0, 0 \rangle$
Blame	$\langle 0, 1 \rangle$	$\langle -1, 0 \rangle$

If, say, there is a 50% probability of guilt then blame would have an expected utility of $\langle -0.5, 0.5 \rangle$ which would be outranked by the $\langle 0, -0.5 \rangle$ expected utility of withholding blame. But if this probability were 90% or above then the possibility of innocence would count as a de minimis risk and would be discounted, in which case the expected utility of blame would be $\langle 0, 1 \rangle$ and would outrank the $\langle 0, -1 \rangle$ expected utility of withholding blame.

Table 6: To blame or not to blame? (multidimensional utilities, partial blame included)

	Person is guilty	Person is innocent
Withhold blame	$\langle 0, -1 \rangle$	$\langle 0, 0 \rangle$
Partially blame	$\langle 0, x \rangle, -1 < x < 1$	$\langle y, 0 \rangle, -1 < y < 0$
Fully blame	$\langle 0, 1 \rangle$	$\langle -1, 0 \rangle$

There are no numbers that we can put in for x and y such that partial blame will be capable of maximising expected utility. If the probability of guilt is below 0.9 then the ranking of the options will be determined by movement in the first dimension, and withholding blame will outrank partial blame. If the probability of guilt is 0.9 or above then the second dimension will take over, and full blame will outrank partial blame. There is no probability window in which partial blame will be the preferred option.