A CONNEXIVE CONDITIONAL

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ABSTRACT: We propose a semantics for a connexive conditional based on the Lewis-Stalnaker conditional. It is a connexive semantics that is both classical and intuitive.

KEYWORDS: connexivity, Lewis-Stalnaker conditional, semantics

1. Introduction

Connexive logic and conditionals are intertwined. The implications of connexive systems have long been thought to be candidates for analysing indicative and subjunctive conditionals (Angell 1962; McCall 1966; 1975; Cantwell 2008; Kapsner and Omori 2017). McCall (2014, 964) says that the logic of causal and subjunctive conditionals is connexive "since 'If X is dropped, it will hit the floor' contradicts 'If X is dropped, it will not hit the floor'." Weiss (2019) puts forth sound and complete connexive logics for conditionals, using an algebraic semantics in the tradition of Nute (1980). However, what is badly missing is an intuitive semantics for a connexive conditional. As Kapsner (2019, 516) recently pointed out, no strongly connexive logic has "an intelligible and well-motivated semantics."

Why is an intuitive semantics for a strongly connexive conditional so hard to come by? One of the reasons is that strongly connexive logics are usually many-valued and it is hard to give some of those values an intuitive meaning. Another reason is that many connexive logics do not validate the principle of classical logic that anything follows from an arbitrary contradiction. The consequence relation of those logics is paraconsistent: the relation is non-trivial even if the premises are inconsistent (Priest et al. 2018). This non-trivial reasoning from inconsistent premises poses at least a challenge for intelligibility (Rudnicki 2021).

Here we propose an intuitive semantics for a connexive conditional. We do so by defining the connexive conditional in terms of a Lewis-Stalnaker conditional. Notably, we remain within the realm of consistent classical semantics for conditionals (in the tradition of Stalnaker (1968), Lewis (1973a), and Chellas (1975)). The connexive semantics we propose is a modal extension of classical logic (Kripke 1963). Unlike many connexive logics, our conditional semantics validates, in particular, the principle of bivalence.

Section 2 outlines what a connexive conditional is and explains why the current contenders for an intuitive semantics for conditionals are not connexive.

Section 3 introduces the idea to restrict the guiding ideas of connexivity to consistent antecedents. Section 4 proposes our semantics for a non-restrictedly connexive conditional. Section 5 concludes the paper.

2. What is a Connexive Conditional?

There are two guiding ideas behind connexive logics. One of them is that no proposition can entail its own negation (Anderson & Belnap 1975; Wansing 2020). This idea can be understood as follows: for no proposition A, $A > \neg A$ is satisfiable, where $>$ expresses a conditional or an implication. The other guiding idea of connexive logic is that if A implies C, then A does not imply $\neg C$. In symbols, if $A > C$, then $A \not> \neg C$.[1]

Nowadays, connexive logics are taken to express the two guiding ideas by validating the following principles.

$$\neg(A > \neg A) \qquad \text{(AT')}$$

$$(A > C) > \neg(A > \neg C) \qquad \text{(BT)}$$

Indeed, a logical system is currently said to be connexive only if AT' and BT are theorems and the conditional is non-symmetric (Wansing 2020).[2] It is worth mentioning that many non-classical connexive logics satisfy AT' and BT, but still violate our guiding ideas. In the logic of Cantwell (2008), for instance, $A > \neg A$ is satisfiable, and so is $A > C$ and $A > \neg C$ at the same time. As Kapsner (2012, 143) puts it, it "seems clear to me that a logic like Cantwell's [...] clearly violates the original intuitions that first prompted the adoption of [AT'] and [BT]." The 'original intuitions' or guiding ideas behind connexive logic are, of course, validated by other non-classical connexive implications, e.g. Angell (1962) and McCall (1966).

The two guiding ideas are highly plausible for an intuitive semantics of conditionals. And yet the mainstream contenders for the semantics of our everyday conditional violate the guiding ideas. Jackson defends that the material implication $\rightarrow$ of classical logic gives the truth conditions of *if* (Jackson 1979; 1980). However, $A \rightarrow \neg A$ is true whenever A is false. On the variably strict semantics for conditionals, $A > \neg A$ is true whenever A is impossible (Stalnaker 1968; Lewis 1973b). And likewise on the strict analysis: where $\Box$ is a necessity operator, $\Box(A \rightarrow \neg A)$ is true whenever A is impossible (Gillies 2007; Kratzer 1989; 2012). Similarly, $A \rightarrow C$ and $A \rightarrow \neg C$ are both true whenever A is false. The Stalnaker-Lewis conditionals $A > C$ and $A > \neg C$

[1] The two guiding ideas roughly correspond to the two intuitions in Kapsner (2012, 142).

[2] Non-symmetry requires that $(A > C) > (C > A)$ is not valid. A logic is said to be connexive iff in addition to AT', BT, and non-symmetry, AT and BT' are theorems of the logic. AT says $\neg(\neg A > A)$, and BT' says $(A > \neg C) > \neg(A > C)$.

are both true whenever A is impossible, and so are the corresponding strict implications.

As we have just seen, the mainstream contenders for the everyday conditional violate the plausible guiding ideas of connexive logic. The first guiding idea speaks decisively against the material implication. Whenever A is false, $A \rightarrow \neg A$ is true. And whenever A is true, $\neg A \rightarrow \neg\neg A$ is true. Whether or not A is true, the first guiding idea is violated.[3] The first guiding idea, taken as a desideratum, challenges the material implication as giving the truth conditions for *if*. But perhaps the material implication is for other reasons the weakest of the current contenders anyways (Starr 2019; Edgington 2020).

3. Restriction to Consistent Antecedents

Why do the variably strict and strict conditionals violate the guiding ideas of connexive logic? The reason seems to be that, from a classical perspective, an inconsistent or contradictory proposition entails everything—including its negation. We make the natural assumption according to which a proposition is inconsistent or contradictory iff it is logically impossible. To be explicit, we say that a proposition A is logically possible iff there is a possible world where A is true. A proposition A is thus impossible iff there is no world at which A is true. Our assumption then says: for each world at which a proposition is evaluated, there is a possible world where the proposition is true iff the proposition is classically consistent. Furthermore, we say that a world which satisfies a proposition A is an A-world. Hence, there is no A-world *only* if A is inconsistent.

Under our assumption, there is an obvious way to save the guiding ideas for the variably strict and strict conditionals. One may restrict the guiding ideas to consistent—or equivalently logically possible—antecedents. The first idea would become: for no logically possible proposition A, $A > \neg A$. The second idea would become: for any logically possible proposition A, if $A > C$, then $A \not> \neg C$. And indeed, the strict and variably strict conditionals validate those restricted principles. $\Box (A \rightarrow \neg A)$ is true at a world iff $\neg A$ is true at *all* worlds. By the restriction, we only look at antecedents for which there is a possible world where the antecedent is true. So there is a world where A is true and thus not all worlds make $\neg A$ true. We have that $\Box (A \rightarrow \neg A)$ is not satisfiable if A is logically possible. As to the second restricted guiding idea: suppose there is an A-world and all worlds are $\neg A \vee C$-worlds. Well then there is an A-world which is a C-world, and so it is not the case that all worlds are $\neg A \vee \neg C$-worlds.

[3] Moreover, one of AT' and AT is violated whether or not A is true.

A variably strict conditional $A \mathrel{\Box\!\!\rightarrow} C$ is true at a world w just in case C is true at all A-worlds most similar to w. The restricted guiding ideas are validated. Whenever there is an A-world, $A \mathrel{\Box\!\!\rightarrow} \neg A$ is false at any world. Moreover, suppose there is an A-world and C is true at all A-worlds most similar to w. Well then there is a most similar A-world at which C is true, and so it is not the case that $\neg C$ is true at all A-worlds most similar to w. So the restricted guiding ideas are validated by variably strict and strict conditionals.[4]

The idea to restrict the guiding ideas of connexive logic is not new. Lenzen (2020), for example, believes that Aristotle and Boethius—the name givers for AT' and BT respectively—intended the theses to apply only to conditionals with consistent antecedents. Lenzen (2019) states the following restricted version of AT':

$$\Diamond A > \neg(A > \neg A) \qquad \text{(LEIB1)}$$

Most variants of variably strict and strict conditionals validate LEIB1, as Lenzen and Kapsner (2019) already pointed out.[5] The latter introduces the restriction to non-contradictory antecedents under the label of *humble connexivity*. But not everyone thinks the connexive principles should be restricted. Restricting AT' and BT to consistent (or satisfiable) antecedents, so claims for example Wansing (2020, 62), makes the "whole enterprise of connexive logic pointless from the standpoint of classical logic."

Are the connexive theses restricted to satisfiable antecedents more intuitive than the original versions? Does a contradictory antecedent entail its own negation? Perhaps. Iacona (2019, 6), for instance, defends that the truth of the conditional "If it is snowing and it is not snowing, then it is snowing" is "not implausible." But, even if so, the truth of $A \wedge \neg A > B$ for any B is hardly intelligible. A contradiction does not help to understand anything. To see this, notice that the conditional "if it is snowing and it is not snowing, then it is snowing and not snowing" may well be true according to Iacona's defense. But how is the world like if it snows and it does not at the same time?

It remains to be seen whether the restricted versions of connexivity will replace the established concept of connexivity. In the meantime, we propose a semantics that puts the restriction into the meaning of the conditional rather than restricting the connexive theses themselves. The result is an intuitive semantics for an unrestrictedly connexive conditional.

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[4] The variably strict and strict conditionals likewise validate AT' and BT.

[5] Lenzen correctly remarks that LEIB1, named after Leibniz, is a theorem of almost all systems of normal modal logic.

4. A Connexive Conditional

We have seen that a variably strict conditional $A \Box\!\!\rightarrow C$ is true at a world w just in case C is true at all A-worlds most similar to w. We modify this semantic clause to obtain a connexive semantics. The main idea is to require for the truth of a conditional that there is an antecedent world. A conditional $A > C$ is true at w iff there is an A-world and C is true at all A-worlds most similar to w. In brief, we define $A > C$ as $\Diamond A \wedge (A \Box\!\!\rightarrow C)$.[6]

The variably strict conditional $A \Box\!\!\rightarrow C$ is automatically true (at any world) when the antecedent A is impossible. By contrast, the conditional $A > C$ is automatically false (at any world) when the antecedent is impossible. If A is (logically) impossible, then there is no A-world, and so no most similar A-world w_A. Likewise, if there is an A-world, there is a most similar A-world. We have baked the restriction to logically possible (or consistent) antecedents into the truth conditions of >. To reiterate, $A > C$ is false at any world if A is logically impossible.

Here is a handy way to state the truth conditions for our conditional. Let $[C]$ abbreviate the set of all C-worlds, and $[W_A]$ the set of all A-worlds most similar to w. We can thus simplify our semantic clause:

$A{>}C$ is true at w iff there is w_A and $[W_A] \subseteq [C]$.

Why is this semantics connexive? A first reason is that > satisfies our guiding ideas. For no proposition A, $A > \neg A$ is satisfiable. Indeed, $A > \neg A$ is true at no world. To see this, consider two cases. First, suppose A is possible. Then there is a most similar A-world w_A, but all w_A satisfy A and thus do not satisfy $\neg A$. In symbols, $[W_A] \nsubseteq [\neg A]$. Second, suppose A is impossible. Then there is no A-world, and a fortiori no most similar A-world w_A. Hence, $A > \neg A$ is false at every world. So AT' is a tautology for >.[7]

The conditional satisfies the other guiding idea as well. Suppose $A > C$ is true at an arbitrary world w. Then there is w_A and $[W_A] \subseteq [C]$. So there is at least one most similar A-world that satisfies C. This implies that not all most similar A-worlds are $\neg C$-worlds. Hence, $A > \neg C$ is false at the arbitrarily chosen world w. A weak form of BT is thus a tautology for >:

[6] Lewis (1973a, 25) and Lewis (1973b, 438) strengthen his more subtle semantics for conditionals in similar ways. Priest (1999, 145-6) strengthens the strict conditional in a similar way and arrives at a semantics for a "simple" connexivist logic. However, he does so in a more complicated way using a "symmetrised account."

[7] By similar reasoning, $\neg A > A$ is false at every world. So AT is also a tautology for >.

$$(A > C) \rightarrow \neg(A > \neg C) \qquad \text{(BTw)}$$

This establishes that our conditional is *connexive* according to Weiss (2019).[8] Moreover, our conditional satisfies the rule form of BT:

$$(A > C) \models \neg(A > \neg C) \qquad (\mathrm{BT}^{r})$$

This makes our conditional also *weakly connexive* according to Wansing and Unterhuber (2019).[9]

As is clear by now, our conditional validates the principle of conditional non-contradiction, also known as Abelard's First Principle:

$$\neg((A > C) \wedge (A > \neg C)) \qquad \text{(ABEL1)}$$

We have illustrated ABEL1 in the introduction by means of quoting McCall.

It should be mentioned that our conditional does not validate BT. The reason is that $A > C$ may be impossible. Then there is no $w_{A>C}$. And so $(A > C) > \neg(A > \neg C)$ is false (at any world).[10] But since our conditional validates the guiding ideas and the other connexive principles, we think that this result rather casts doubt on BT than our conditional. BT is too demanding for expressing the second guiding idea.

Finally, and to support the point of the last paragraph, our conditional satisfies BT when the main connective is replaced by a variably strict conditional which may be vacuously true:

$$(A > C) \mathrel{\Box\!\!\rightarrow} \neg(A > \neg C) \qquad \text{(VBT)}$$

Suppose $A > C$ is impossible. Then there is no $w_{A>C}$. Hence, $(A > C) \mathrel{\Box\!\!\rightarrow} \neg(A > \neg C)$ is vacuously true.[11]

5. Conclusion

We have proposed a semantics for a connexive conditional. The conditional $A > C$ is true at a world w just in case there is an A-world *and* C is true at all A-worlds most similar to w. This strengthening of the Lewis-Stalnaker conditional validates the two guiding ideas behind connexive logics, as well as the connexive theses AT, AT', BTw, BTw', BT^{r}, $\mathrm{BT}^{r\prime}$, and ABEL1. Moreover, our conditional validates VBT, a version of BT where the main connective is a variably strict conditional. Indeed, the variably

[8] Again, by similar reasoning, $A > C$ is false at any world if $A > \neg C$ is true there. So BTw', $(A > \neg C) \rightarrow \neg(A > C)$, is also a tautology for $>$.

[9] Our conditional satisfies the rule form of BT' as well.

[10] BT' is also no tautology for our conditional.

[11] VBT' is valid for our conditional as well.

strict conditional $\Box\!\!\rightarrow$ and our conditional $>$ are interdefinable using only Boolean connectives. Where $\bot$ stands for an arbitrary contradiction, we have

$$A > C =^{\mathrm{df}} \neg(A \Box\!\!\rightarrow \bot) \wedge (A \Box\!\!\rightarrow C), \text{ and}$$

$$A \Box\!\!\rightarrow C =^{\mathrm{df}} \neg(A > A) \vee (A > C).$$

Unlike others, we have not restricted the connexive theses to consistent or possible antecedents. Rather we have baked this restriction into the semantics of our conditional. The result is a semantics for (causal) conditionals which is as intuitive as the Lewis-Stalnaker semantics when the antecedent is (logically) possible. When the antecedent is (logically) impossible, our conditional is automatically false. Intuitively, the conditional "If the vase is dropped and not dropped at the same time, it will hit the floor" is at least not true. And the conditional is not exactly intelligible. For all practical purposes, we may consider it false.[12]

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[12] Is the negation of "If the vase is dropped and not dropped at the same time, it will hit the floor" true? On our semantics, 'it is not the case that if the vase is dropped and not, it will hit the floor' is true. But we will never say such conditionals in an ordinary context.

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